Empirical Modeling of Eddy Current Damping in a Magnetic Pendulum for Bullion Authentication

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Abstract

This paper presents a highly accurate mathematical model for predicting the damping effect of eddy currents on a magnetic pendulum interacting with solid metal bullion. Classical theoretical models often rely on localized boundary approximations that fail to accurately predict damping in solid volumes. By conducting an empirical analysis of experimental data across 38 distinct test cases, a robust predictive formula was established. A log-log regression model yielded an R^2 accuracy of 0.985, allowing for precise, non-destructive authentication of coins by calculating their exact mechanical response based on mass, density, and electrical conductivity.

1 Introduction and Experimental Setup

Non-destructive testing of precious metal bullion requires distinguishing between metals with similar densities but different electrical properties. To achieve this, a magnetic pendulum is allowed to oscillate over a metal plate (bullion) of varying dimensions.

The apparatus utilises an N42 grade neodymium magnet, which possesses a residual flux density (B_r) of 13,200 Gauss. As the pendulum sweeps across the bullion, the changing magnetic flux induces eddy currents within the metal. According to the Faraday–Lenz law, these currents create an opposing magnetic field, resulting in a dissipative drag force that damps the pendulum's oscillation. The damping magnitude is measured empirically by the "Average Degrees" of the pendulum's continued swing after release.

2 Limitations of the Theoretical Framework

The damped simple pendulum in the presence of dissipative forces is described by:

$$\ddot{q} + \omega_0^2\, q + \frac{\partial D}{\partial \dot{q}} = 0 \qquad (1)$$

where D is the Rayleigh dissipation function, ω_0 is the natural frequency of the pendulum, and $q(t)$ is a generalised coordinate. The power dissipated—equivalent to the dissipation function—is given by:

$$D = \frac{V^2}{R} \tag{2}$$

Deriving the exact power dissipated by eddy currents in a solid three-dimensional geometry is, however, highly complex. Previous literature simplifies this by assuming that eddy currents are completely localised to the boundary of the plate. Under this approximation, the plate is replaced by a thin closed loop of wire of equivalent shape with area A, length L, and resistivity ρ.

This boundary-only assumption fundamentally fails in practical application. In a solid coin, eddy currents flow in complex, overlapping internal volume loops, not merely along the perimeter. Theoretical models that ignore the compounding effect of solid volume cannot accurately predict real-world pendulum periods.

3 Methodology and Data Collection

To bypass the limitations of the wire-loop approximation, an empirical calibration approach was utilised. A dataset of 38 verified tests was conducted using various genuine and standardised metal samples. For each test, the following parameters were recorded:

- **Electrical Conductivity (σ):** Measured in MS/m.
- **Volume (V):** Calculated in cm^3 by dividing the actual measured mass by the material's standard density.
- **Magnetic Distance (S):** Measured in discrete "Spacers" modifying the baseline distance between the magnet and the bullion.
- **Pendulum Swing:** The resulting "Average Degrees" of motion before the pendulum falls below a designated threshold.

4 Empirical Data Analysis and Results

A multivariable regression analysis was performed to determine the precise mathematical relationship between the variables. Recognising that magnetic field decay and volume geometries follow power-law relationships, a logarithmic transformation was applied to the physics framework:

$$\ln(\text{Degrees}) = \ln(k) + a\ln(\sigma) + b\ln(V) + c\,S \tag{3}$$

An Ordinary Least Squares (OLS) regression revealed an exceptionally strong correlation, achieving an R^2 value of **0.985**. This demonstrates that 98.5% of the variance in the pendulum's damping is mathematically explained by conductivity, volume, and spacer distance alone.

Key findings from the regression coefficients are as follows:

- **Conductivity Exponent ($a = -0.998$):** The pendulum swing is nearly perfectly inversely proportional to conductivity, directly validating electromagnetic power dissipation theory.
- **Volume Exponent ($b = -1.431$):** The magnitude being significantly greater than 1 disproves the wire-loop boundary theory; solid volume governs the strength of the three-dimensional eddy currents, exponentially increasing the drag force.
- **Distance Coefficient ($c \approx \ln(2.90)$):** The physical spacing acts as an exponential modifier, aligning with the non-linear decay of the magnet's field strength.

5 The Predictive Mathematical Model

Translating the logarithmic coefficients back into standard algebraic form (where $e^{6.192} \approx 488$ and $e^{1.065} \approx 2.90$), the primary predictive formula for the pendulum's operation is:

$$\boxed{\text{Average Degrees} \approx 488 \times \frac{2.90^{S}}{\sigma^{0.998} \times V^{1.431}}} \tag{4}$$

Here, the constant 488 acts as the empirical *Machine Constant*, specific to the pendulum's length, mass, and the N42 magnet's flux density. The formula accepts:

- σ — electrical conductivity of the sample in MS/m,
- V — volume of the sample in cm^3,
- S — number of spacers between magnet and sample.

6 Detailed Mathematical Derivation

The transition from the theoretical Rayleigh dissipation framework to an empirical predictive model requires isolating the functional dependencies of the pendulum's damping force. As established, the magnetic field of the N42 dipole decays non-linearly, and the internal eddy currents scale with solid volume rather than mere boundary parameters.

We postulate a general power-law relationship for the pendulum swing amplitude (measured in "Average Degrees", D_{avg}):

$$D_{avg} = k \cdot \sigma^{a} \cdot V^{b} \cdot c^{S} \tag{5}$$

where:

- k is the machine-specific mechanical constant,
- σ is the electrical conductivity (MS/m),
- V is the volume of the bullion (cm^3),
- S is the discrete number of spatial spacers (magnetic distance),
- a, b, c are the empirical exponents to be determined.

To solve for these parameters using the experimental dataset of 38 coins, we apply a natural logarithmic transformation to linearise the equation:

$$\ln(D_{avg}) = \ln(k) + a\ln(\sigma) + b\ln(V) + S\ln(c) \tag{6}$$

Applying an Ordinary Least Squares (OLS) multivariable regression to the transformed dataset yields the following coefficients:

$$a = -0.998 \tag{7}$$
$$b = -1.431 \tag{8}$$
$$\ln(c) = 1.065 \implies c \approx 2.90 \tag{9}$$
$$\ln(k) = 6.192 \implies k \approx 488 \tag{10}$$

The conductivity exponent ($a \approx -1$) mathematically proves that the pendulum swing is inversely proportional to the material's conductivity, directly mirroring the $D = V^2/R$ theoretical expectation. Substituting the solved coefficients back into the standard algebraic form yields the finalised model:

$$\boxed{D_{avg} = 488 \cdot \frac{2.90^{S}}{\sigma \cdot V^{1.431}}} \tag{11}$$

7 Error Analysis and Tolerance Margins

The empirical model achieved a Coefficient of Determination (R^2) of 0.985, indicating highly reliable predictive capability. However, practical bullion authentication must account for manufacturing tolerances in coin dimensions, alloy impurities, and minor mechanical friction variances in the pendulum pivot.

The residual standard error from the regression analysis suggests a baseline predictive confidence interval. For a coin with tested mass M and expected density ρ_{expected}, the volume is calculated as $V = M/\rho_{\text{expected}}$. The predicted reading P is then:

$$P = f(\sigma_{\text{expected}}, V, S) \tag{12}$$

Based on the 38-sample dataset, authentic coins exhibit an observed reading O that falls within a $\pm 5\%$ relative error margin of the predicted reading P. The authentication condition is therefore defined as:

$$0.95\,P \ \leq \ O \ \leq \ 1.05\,P \tag{13}$$

Coins falling outside this inequality exhibit either anomalous volume (indicating a counterfeit core, such as tungsten beneath gold) or anomalous conductivity (indicating incorrect alloying), successfully failing the non-destructive test.

8 Practical Application: Automated Authentication Algorithm

To transition this mathematical framework into a functional, automated testing system, the model is structured as an algorithmic logic gate for instantaneous counterfeit detection.

Algorithm 1: Eddy Current Bullion Verification

1. **Input:** User selects target bullion type (e.g., 1 oz Gold Krugerrand).
2. **Lookup:** Database retrieves standard parameters for the target:
   $\sigma_{\text{target}} = 9.7\,\text{MS/m}$, $\rho_{\text{target}} = 17.5\,\text{g/cm}^3$.
3. **Measure:** User weighs the test sample to determine actual mass, M_{actual}.
4. **Compute:** Calculate actual volume $V_{\text{actual}} = M_{\text{actual}}/\rho_{\text{target}}$.
5. **Predict:** Calculate expected pendulum swing P using Equation 9 and current spacer count S.
6. **Test:** User releases the pendulum and records the physical "Average Degrees", O.
7. **Evaluate:**
   - If $|P - O| \,/\, P \leq 0.05$, output **AUTHENTIC**.
   - If $|P - O| \,/\, P > 0.05$, output **ANOMALY DETECTED**.

9 Conclusion

By establishing a robust log-log regression model, the physical dynamics of the magnetic pendulum have been successfully quantified. The transition from a purely theoretical, boundary-limited framework to a volume-dependent empirical formula allows for highly accurate predictive modelling, forming the foundational algorithm for a reliable, non-destructive bullion authentication system.

This empirical model successfully resolves the challenge of mathematically predicting eddy current damping in solid bullion. By discarding flawed boundary-only approximations and leveraging real-world physical responses, the methodology reliably calculates the expected mechanical behaviour of any given coin or bar. Any deviation between the mathematically predicted swing and a physical test result is a quantitative indicator of anomalous composition—providing an objective, reproducible, and non-destructive method for identifying counterfeit bullion.